QANT 620-M01 Fall 2013
A1 - Good

Goal Program Case 7.1

QANT 620-M01

Professor: Dr. Tibrewala

4,5,4 Michall Abaev 0887745
4,5,4 Dong Liang 0846163
4,5,4 Hong Xu 0839296

Introduction:

Removing snow in Montreal, Canada is extremely important and very expensive. Montreal setup a time-efficient system for the removal of snow. The city is divided into several sectors and snow removal operations are carried out concurrently in each sector.

Different types of disposal sites can accommodate different amounts of snow due to the physical size of the disposal facility. Below are the annual capacities for five different snow disposal sites in 1,000's of cubic meters:

	Disposal Site				
	1	2	3	4	5
Capacity	350	250	500	400	200

The disposal sites are equipped to remove different amounts of contaminants from the snow they receive. Below shows the percentage of contaminants that can be removed from the snow delivered to each disposal site:

	Disposal Site				
	1	2	3	4	5
Contaminant removed	30%	40%	20%	70%	50%

Montreal uses the straight-line distance between the center of each sector to each of the various disposal sites as an approximation of the cost involved in transporting snow between these locations. Below shows the distances in kilometers for ten sectors in Montreal:

Distance	Site 1	Site 2	Site 3	Site 4	Site 5
Sector 1	3.4	1.4	4.9	7.4	9.3
Sector 2	2.4	2.1	8.3	9.1	8.8
Sector 3	1.4	2.9	3.7	9.4	8.6
Sector 4	2.6	3.6	4.5	8.2	8.9
Sector 5	1.5	3.1	2.1	7.9	8.8
Sector 6	4.2	4.9	6.5	7.7	6.1
Sector 7	4.8	6.2	9.9	6.2	5.7
Sector 8	5.4	6	5.2	7.6	4.9
Sector 9	3.1	4.1	6.6	7.5	7.2
Sector 10	3.2	6.5	7.1	6	8.3

The table below estimates the snow removal requirements in 1000s of cubic meters for each sector in the coming year:

Estimated Annual Snow Removal Requirements									
1	2	3	4	5	6	7	8	9	10
153	152	154	138	127	129	111	110	130	135

a. If Montreal wants to pursue the objective of minimizing the distance the snow must be moved (and therefore the cost of removing snow), how much snow should it plan to move from each sector to each disposal site?

   i. Objective: Here Montreal wants to minimize the distance snow must be moved. To pursue such an objective we need to know how much snow it should plan to move from each sector to each disposal site.

      1. First we illustrate our variables:

         a. Xij=1 if we assign sector i=1,2,3,.....,10 to site j=1,2,3,..5 and 0 otherwise

         b. We minimize: $(3.4_{x11}+1.4_{x12}+4.9_{x13}+7.4_{x14}+9.3_{x15})+$ $(2.4_{x21}+2.1_{x22}+8.3_{x23}+9.1_{x24}+8.8_{x25})$.................$+(3.2_{x101}+6.5_{x102}+7.1_{x103}+6_{x104}+.3_{x105})$

         c. X11+X12+X13+X14+X15=153

         d. X21+X22+X23+X24+X25=152

         e. ……….

         f. X101+X102+X103+X104+X105=135

         g. X11+X21+…+X101<=350

         h. ………

         i. X15+X25+…+X105<=200

   ii. Below we will show you how we found how much snow Montreal should plan to move from each sector to each disposal site through excel:

sector	1	2	3	4	5		
1	3.4	1.4	4.9	7.4	9.3		
2	2.4	2.1	8.3	9.1	8.8		
3	1.4	2.9	3.7	9.4	8.6		
4	2.6	3.6	4.5	8.2	8.9		
5	1.5	3.1	2.1	7.9	8.8		
6	4.2	4.9	6.5	7.7	6.1		
7	4.8	6.2	9.9	6.2	5.7		
8	5.4	6	5.2	7.6	4.9		
9	3.1	4.1	6.6	7.5	7.2		
10	3.2	6.5	7.1	6	8.3		

sector	1	2	3	4	5	shipped	available
1	0	153	0	0	0	153	153
2	55	97	0	0	0	152	152
3	29	0	125	0	0	154	154
4	0	0	138	0	0	138	138
5	0	0	127	0	0	127	127
6	1	0	0	0	128	129	129
7	0	0	0	39	72	111	111
8	0	0	110	0	0	110	110
9	130	0	0	0	0	130	130
10	135	0	0	0	0	135	135
shipped	350	250	500	39	200		
capacity	350	250	500	400	200		
nant remov	30%	40%	20%	70%	50%		
minant re	432.3						
distace	4784.9						
(a)	4784.9						
(b)	167471.5						

Therefore, Montreal should move 153 cubic meters of snow from sector 1 to site 2, 55 cubic meters of snow from sector 2 to site 1 and 97 cubic meters of snow from sector 2 to site 2, and so on for the other sectors.

b. If it costs $35 to move 1000 cubic meters of snow one kilometer, how much should Montreal plan on spending on the transportation for the removal of snow?

If it costs $35 to move 1000 cubic meters of snow one kilometer, Montreal should plan on spending $167,471.50 on the transportation for the removal of snow.

a. We minimize: $((3.4_{x11}+1.4_{x12}+4.9_{x13}+7.4_{x14}+9.3_{x15})+$

$(2.4_{x21}+2.1_{x22}+8.3_{x23}+9.1_{x24}+8.8_{x25})\ldots\ldots\ldots\ldots+$

$(3.2_{x101}+6.5_{x102}+7.1_{x103}+6_{x104}+.3_{x105}))*\$35$

$=4784.9*\$35$

$=\$167471.5$

c. If Montreal wants to pursue the objective of maximizing the amount of contaminant that is removed from transported snow, how much snow should it plan to move from each sector to each disposal site and what transportation cost is associated with this solution?

Objective: Here Montreal wants to maximize the amount of contaminant that is removed from transported snow.

First, we set our variables, B2:F11.

Then, we set constraints.

1. G2:G11=H2:H11
2. B12:F12=<B13:F13

We select Simplex LP as our solving method.

	A	B	C	D	E	F	G	H
1	sector	1	2	3	4	5	shipped	available
2	1	0	0	0	153	0	153	153
3	2	13	0	139	0	0	152	152
4	3	83	0	0	71	0	154	154
5	4	0	138	0	0	0	138	138
6	5	15	112	0	0	0	127	127
7	6	129	0	0	0	0	129	129
8	7	0	0	0	111	0	111	111
9	8	110	0	0	0	0	110	110
10	9	0	0	0	0	130	130	130
11	10	0	0	0	65	70	135	135
12	shipped	350	250	139	400	200		
13	capacity	350	250	500	400	200		
14	inant remo\	30%	40%	20%	70%	50%		
15	minant rem	612.8						

sector	1	2	3	4	5		
1	3.4	1.4	4.9	7.4	9.3		
2	2.4	2.1	8.3	9.1	8.8		
3	1.4	2.9	3.7	9.4	8.6		
4	2.6	3.6	4.5	8.2	8.9		
5	1.5	3.1	2.1	7.9	8.8		
6	4.2	4.9	6.5	7.7	6.1		
7	4.8	6.2	9.9	6.2	5.7		
8	5.4	6	5.2	7.6	4.9		
9	3.1	4.1	6.6	7.5	7.2		
10	3.2	6.5	7.1	6	8.3		
sector	1	2	3	4	5	shipped	available
1	0	0	0	153	0	153	153
2	13	0	139	0	0	152	152
3	83	0	0	71	0	154	154
4	0	138	0	0	0	138	138
5	15	112	0	0	0	127	127
6	129	0	0	0	0	129	129
7	0	0	0	111	0	111	111
8	110	0	0	0	0	110	110
9	0	0	0	0	130	130	130
10	0	0	0	65	70	135	135
shipped	350	250	139	400	200		
capacity	350	250	500	400	200		
inant remove	30%	40%	20%	70%	50%		
aminant rem	612.8						
distance	7698.2						
cost	269437						

The cost is 7698.2*$35=$269437

d. Suppose Montreal wants to minimize the maximum percentage deviation from the optimal value for each of the two objectives mentioned earlier. What is the optimal solution and how far is each objective function from its optimal value?

Multi-objective optimization is an area of multiple criteria decision making, that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. To solve this problem we will be utilizing this decision making model.

We have minimized our maximum percentage deviation and we received 5.407 % deviation.

Objective: Here Montreal wants to maximize the amount of contaminant that is removed from transported snow.

First, we set our variables, B14:F23.

Then, we set constraints.

1. G14:G23=H14:H23
2. B24:F24=<B25:F25
3. B14:F23>=0
4. F29:F30=<D32

sector	1	2	3	4	5		
1	3.4	1.4	4.9	7.4	9.3		
2	2.4	2.1	8.3	9.1	8.8		
3	1.4	2.9	3.7	9.4	8.6		
4	2.6	3.6	4.5	8.2	8.9		
5	1.5	3.1	2.1	7.9	8.8		
6	4.2	4.9	6.5	7.7	6.1		
7	4.8	6.2	9.9	6.2	5.7		
8	5.4	6	5.2	7.6	4.9		
9	3.1	4.1	6.6	7.5	7.2		
10	3.2	6.5	7.1	6	8.3		
sector	1	2	3	4	5	shipped	available
1	0	153	0	0	0	153	153
2	55	97	0	0	0	152	152
3	154	0	0	0	0	154	154
4	59.73025	0	78.26975	0	0	138	138
5	0	0	127	0	0	127	127
6	0	0	0	39	90	129	129
7	0	0	0	111	0	111	111
8	0	0	0	0	110	110	110
9	81.26975	0	0	48.73025	0	130	130
10	0	0	0	135	0	135	135
shipped	350	250	205.2697	333.7303	200		
capacity	350	250	500	400	200		
nant remov	30%	40%	20%	70%	50%		
	actual	target	%deviatio	weight	weighted deviation		
distance	5043.626	4784.9	5.407127	1	5.407127		
mmnant rem	579.6651	612.8	5.407127	1	5.407127		
mmnant rem	579.6651	612.8	5.407127	5	27.03563		
		Q d	5.407127				
		Q e	27.03563				

The percentage deviation is the same. So both solutions are optimal.

e. Suppose the removal of contaminants is regarded as five times more important than transportation cost minimization. What solution minimizes the maximum weighted percentage deviation for each objective? How far is each objective from its optimal value?

In the Excel, we change the weight 1 into 5, and get a percentage deviation is 27.03563%. So the optimal solution is to minimize the contaminants.

f. What other suggestions might you have for Montreal as it attempts to deal with these two conflicting objectives?

1. Make a decision involving maximizing the amount of contaminants that is removed and minimizing the distance the snow must be removed. This decision needs to be flexible according to the reality. Calculate the cost the contaminants not removed takes and compare it with the cost of transportation. The cost taken by the contaminants not removed is not only the money needs to be spent on curbing environmental pollution, but also the cost of the bad influence in the next years. The ecological damage, masses of complaints, public health and so on should be in the consideration.

2. Increase the disposal sites. One of reasons of the high cost of transportation is the lack of disposal sites. More disposal sites means shortening the distance between the sectors and sites and that's shortening the cost of transportation. The government can invest more capital in solving the contaminants.

3. Improve the technique of deicing chemicals. The improved technique of deicing chemicals can make the pollution less. It means to shorten the cost of controlling contaminants, and shorten the deviation of transportation to be closer to optimal decision.

PS: At first, we try to put one constraint that B12:F12 = Binary. In the first form(yellow area). This means there are only 10 trucks to remove snow. But in this case there is nothing mentioning the number of trucks. So we assume that there are unlimited trucks.